Physics 110
Spring 2006
Rotational Dynamics – Their Solutions

1. An object starts from rest and rotates counterclockwise with constant angular acceleration and reaches an angular speed of 12 rad/s in 3 s.
   a. What is the angular acceleration of the wheel?
   b. What is the angle (in radians) through which the object rotates in this time?

   a. $\omega_f = \omega_i + \alpha t \rightarrow \alpha = \dfrac{\omega_f}{t} = \dfrac{12\frac{rad}{s}}{3s} = 4\frac{rad}{s^2}$

   b. $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\times 4\frac{rad}{s^2}\times(3s)^2 = 18rad$

2. A wheel 2.0 m in diameter lies in a vertical plane and rotates with a constant angular acceleration of 4 rad/s². The wheel starts at rest at t = 0 and the radius vector of point P on the rim makes an angle of 57.3° with the horizontal at this time. At t = 2.0 s, find
   a. the angular speed of the wheel.
   b. the linear speed and total acceleration of the point P.
   c. the angular position of the point P.

   a. $\omega_f = \omega_i + \alpha t = \left(4\frac{rad}{s^2}\right)(2s) = 8\frac{rad}{s}$

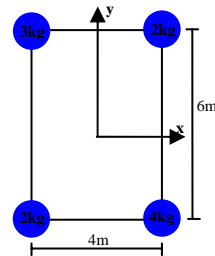   b. $v = r\omega = 1m\times 8\frac{rad}{s} = 8\frac{m}{s}$

   $a_t = r\alpha = 1m\left(4\frac{rad}{s^2}\right) = 4\frac{m}{s^2}$ tangentially

   $a_r = \dfrac{v^2}{R} = r\omega^2 = 1m\times\left(8\frac{rad}{s}\right)^2 = 64\frac{m}{s^2}$

   $a_{total} = \sqrt{a_t^2 + a_r^2} \ @\ \phi = \tan^{-1}\left(\dfrac{a_t}{a_r}\right) \rightarrow 64.1\frac{m}{s^2}\ @\ 3.58^o$

   c. $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = 1rad + \frac{1}{2}\times 4\frac{rad}{s^2}\times(2s)^2 = 9rad$

3. Four point particles of various masses are shown on the right. These masses are connected by massless rods and the system is spun about the origin (located at the center of the rectangle) at 6 rad/s in the x-y plane.
   a. What is the moment of inertia of the system about the z-axis (perpendicular to the x-y plane?)
   b. What is the rotational kinetic energy of the system?
   c. What is the magnitude of the angular momentum of the system about the z-axis?

a. $I_{total} = \sum_i m_i r_i^2 = r^2 \sum_i m_i,$ since all masses are equidistant from the origin.

$$\therefore r_1 = r_2 = r_3 = r_4 = \sqrt{x_1^2 + y_1^2} = \sqrt{(2m)^2 + (3m)^2} = \sqrt{13}m$$

$$I_{total} = \left(\sqrt{13}m\right)^2 (3kg + 2kg + 2kg + 4kg) = 143kgm^2$$

b. $KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(143kgm^2\right)\left(6\frac{rad}{s}\right)^2 = 2574J$

c. $L = I\omega = \left(143kgm^2\right)\left(6\frac{rad}{s}\right) = 858\frac{kgm}{s}$

4. A model airplane having a mass of 0.750kg is tethered by a wire so that it flies in a circle of radius 30m. The airplane engine provides a net thrust of 0.80 N perpendicular to the tethering wire.
    a. What is the torque that the net thrust produces about the center of the circle?
    b. What is the angular acceleration of the airplane when it is in level flight?
   c. Find the linear acceleration of the airplane tangent to its flight path.

a. $\tau = rF_{Thrust} \sin\theta = 30m \times 0.8N = 24Nm$

b. $\tau = I\alpha \rightarrow \alpha = \dfrac{\tau}{I} = \dfrac{24Nm}{675kgm^2} = 0.036\frac{rad}{s^2}$

    where $I_{plane} = MR^2 = 0.75kg \times (30m)^2 = 675kgm^2$

c. $a = r\alpha = 30m \times 0.036\frac{rad}{s^2} = 1.07\frac{m}{s^2}$

5. The apparatus shown below is used to experimentally determine the moment of inertia of irregularly shaped objects. We have a mass m suspended by a cord wound around a spool of radius r, forming part of a turntable supporting the object. When the mass is released from rest, it descends through a distance h, acquiring a speed v. What is the moment of inertia I of the equipment and the turntable?
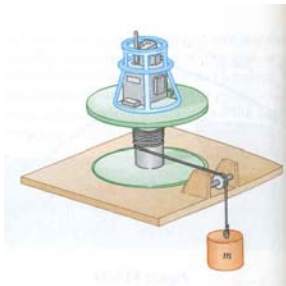

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \rightarrow m_{hanging}gh = \frac{1}{2}m_{hanging}v^2 + \frac{1}{2}I\left(\frac{v}{r_{shaft}}\right)^2$$

$$\therefore I = \left[\frac{2gh - v^2}{v^2}\right]m_{hanging}r_{shaft}^2$$

Note the quantity in brackets is dimensionless – this is the experimental method for determining the constant that appears in the expression for the moment of inertial of this irregularly shaped object.

6. A tennis ball is a hollow sphere with a thin wall. It is set rolling without slipping at 4.03m/s on the horizontal section of track as shown below. It rolls around the inside of a vertical circular loop 90cm in diameter and finally leaves the track at a point 20cm below the horizontal section.

   a. What is the speed at the top of the loop?
   b. What is the centripetal acceleration of the ball?
   c. Does the ball remain in contact with the track? Explain your answer based on your result to part b.
   d. Suppose that static friction between the ball and track was negligible so that the ball slid rather than rolled. Would it speed be higher, lower or the same at the top of the loop? Support your answer with a calculation!
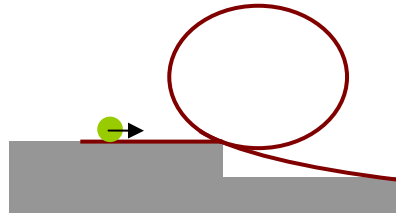
a. $mgh_i + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 = mgh_f + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

$$\rightarrow \sqrt{\frac{2g[h_i - h_f] + \left[1 + \frac{I}{MR_{ball}^2}\right]v_i^2}{\left[1 + \frac{I}{MR_{ball}^2}\right]}} = \sqrt{\frac{2 \times 9.8\frac{m}{s^2}[0.20m - 1.10m] + \left[1 + \frac{\frac{2}{3}MR_{ball}^2}{MR_{ball}^2}\right](4.03\frac{m}{s})^2}{\left[1 + \frac{\frac{2}{3}MR_{ball}^2}{MR_{ball}^2}\right]}}$$

$= 2.39\frac{m}{s} = v_f$

b. $a_c = \frac{v^2}{R} = \frac{(2.39\frac{m}{s})^2}{0.45m} = 12.57\frac{m}{s^2}$

c. Since the a_c is greater than g, the ball must be in contact with the track, with the track pushing down on the ball.

d. $\omega_i = \omega_f = 0$ ball does not roll.

$\therefore \sqrt{2g[h_i - h_f] + v_i^2} = \sqrt{2 \times 9.8\frac{m}{s^2}[0.20m - 1.10m] + (4.03\frac{m}{s})^2} = \sqrt{-1.40\frac{m^2}{s^2}}$

which is non - physical. Therefore the ball doesn't even make it around the loop.

7. A cylinder of moment of inertia I_1 rotates about a vertical frictionless axle with angular velocity ω_i. A second cylinder that has moment of inertia I_2 and initially not rotating is dropped onto the 1st cylinder. Since friction exists between the two surfaces of the cylinders, they eventually reach the same final angular speed, ω_f.

   a. What is the expression for the magnitude of ω_f?
   b. Show that the kinetic energy of the system decreases in this interaction and calculate the ratio of the final rotational energy to the initial rotational energy.

    c. Why does the kinetic energy of the system decrease?

a. $I_1\omega_i = (I_1 + I_2)\omega_f \rightarrow \omega_f = \dfrac{I_1\omega_i}{(I_1 + I_2)}$

b. $KE_i = \frac{1}{2}I_1\omega_i^2; \quad KE_i = \frac{1}{2}(I_1 + I_2)\omega_f^2;$

         $ratio = \dfrac{\frac{1}{2}(I_1 + I_2)\omega_f^2}{\frac{1}{2}I_1\omega_i^2} = \dfrac{I_1}{I_1 + I_2}$ using ω_f from above.

c.   Since KE is not conserved due to friction bringing the two masses to the same final rotational speed, the final KE is less than the initial KE.

8. A student sits on a freely rotating stool holding two weights, each of which has a mass of 3.0kg. When the student's arms are fully extended horizontally, the weights are 1.0m from the axis of rotation and the system rotates with an angular speed of 0.75 rad/s. The moment of inertia of the student and the stool is 3.0 kgm^2 and is assumed to be constant. The student pulls the weights inward horizontally to a position 0.30m from the rotation axis.
   a. What is the new angular speed?
   b. What are the initial and final kinetic energies of the student?
   c. If your answers to part b are not the same number, what happened in the system in order for there to be an increase or decrease in the angular velocity?

a. $I_i\omega_i = I_f\omega_f; \quad I_i = I_{stool} + 2I_{weights}; \ I_f = I_{stool} + 2I_{new,weights}$

         $\therefore \omega_f = \left(\dfrac{I_{stool} + 2I_{weights}}{I_{stool} + 2I_{new,weights}}\right)\omega_i = \left(\dfrac{3kgm^2 + 2(3kg \times (1m)^2)}{3kgm^2 + 2(3kg \times (0.3m)^2)}\right)0.75\frac{rad}{s} = 1.91\frac{rad}{s}$

b. $KE_i = \frac{1}{2}I_i\omega_i^2 = 2.53J$

         $KE_f = \frac{1}{2}I_f\omega_f^2 = 6.46J$

c.   The answers are not the same since it takes work to bring the masses in closer to the body and this work manifests itself as an increase in the rotational KE of the system.

9. The pumpkin on the Nott revisited! Suppose that the Nott Memorial is topped with an approximately hemispherical dome of radius R. Somehow an individual has balanced a spherical pumpkin at the top to the dome at an angle of $\theta_i = 0°$ with the vertical. Suppose that a gust of wind starts the pumpkin rolling from rest. It loses contact with the dome when the line from the center of the hemispherical dome to the pumpkin makes a certain angle with respect to the vertical. At what angle does this happen? Compare your results with those of problem #2 in the work and energy packet.

$$mgh_i = \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f \quad \text{where } h_i = R, \quad h_f = R\cos\theta, \quad I = \tfrac{2}{5}mR^2$$

$$\therefore mgR = \tfrac{1}{2}mgR\cos\theta + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\frac{gR\cos\theta}{R^2} + mgR\cos\theta$$

where $F_N = 0$ when pumpkin leaves the roof. This gives

$$v^2 = Rg\cos\theta$$

$$\therefore 1 = \left(\tfrac{1}{2} + \tfrac{1}{5} + 1\right)\cos\theta \rightarrow \theta = \cos^{-1}\left(\frac{1}{1.7}\right) = 54^o$$

When we include rolling motion for the pumpkin, the pumpkin slides farther before leaving the surface.